Bridging the research-practice gap – a panel report

João Pedro da Ponte[1], Mario Sánchez Aguilar[2], Naďa Vondrová[3], Stefan Zehetmeier[4], Sarah Seleznyov[5] and Jorryt van Bommel[6]

[1]Instituto de Educação da Universidade de Lisboa, Portugal; jpponte@ie.ulisboa.pt

[2]Instituto Politecnico Nacional, Mexico

[3]Faculty of Education, Charles University, Czech Republic

[4]University of Klagenfurt, Austria

[5]Vrije Universiteit Amsterdam, The Netherlands

[6]Karlstad University, Karlstad, Sweden

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Introduction

Mathematics education is primarily a discipline that aims to study the processes that take place in the teaching and learning of mathematics and also in teacher education, in order to improve them. There is no shortage of issues regarding these processes: Why do so many students have difficulties in learning mathematics concepts, representations, and procedures? Why do many students dislike mathematics and work in mathematics? How might development processes be more effective in supporting teachers in their work? What is the real take-up of findings, perspectives and materials from mathematics education researchers by stakeholders as school administrators and policy decision makers? These issues provided the starting point for this panel at CERME13.

The relationship between research and practice (as well as between theory and practice) is a longstanding issue in mathematics education. Many initiatives and events have been devoted to it, such as a mini-conference within ICME 5 (1984) with the participation of scholars such as Erich Wittmann, Leone Burton and Guy Brousseau and the series of events under the name Conference for the Systematic Cooperation between Theory and Practice in Mathematics held between 1984 and 1994. The relationship between research and practice is, indeed, a major issue for CERME, given its interest in having a real impact in improving mathematics education.

One important perspective about this issue that developed in recent years is that the relationship between research (that is, mathematics education researchers and knowledge derived from their work) and practice (that is, the work of mathematics teachers, curriculum developers, teacher educators and many others) is not just unidirectional, namely applying results from research in the field of practice. It also involves assessing the needs of practice and identifying the modes of acting and developing practice to consider when planning, conducting and disseminating research findings. Another important perspective is that the actors in the field, teachers and researchers, may work collaboratively, and often mix their roles, as teachers become involved in research and researchers become involved in teaching.

The participants on the panel come from different parts of Europe (North, Centre, East) and, also, from outside Europe (Mexico), representing different research interests and traditions. There is a large number of issues regarding the relationship between research and practice, and this panel began by addressing the following questions, building on the participants' experiences:

1. What progress are we making? Why is it so difficult to make real progress on this issue?
2. How can we disseminate research findings to practice? How can we translate findings into practical knowledge?
3. What suitable research methods have been proposed to bridge the practice-research gap, such as co-design, action research, design-based research?
4. Are lesson study and professional learning communities effective ways of strengthening this relationship? And what about other forms of teacher education?
5. How can we enable teachers to participate in research? How can we work with teachers as researchers?
6. How should we write for teachers and how can we encourage them to read? What other forms of communication may be effective in reaching teachers?

Short summaries of each of the panel participants' presentations follow this introduction. Sanchez Aguilar presents the research-practice gap as an implementability problem. A case on the use of genetic constructivism in mathematics to bridge the gap is described by Vondrová. From a methodological stance, Zehetmeier suggests Action Research as a method to bridge the theory-practice gap, whereas Seleznyov presents the case of lesson study in the U.K. Finally, van Bommel describes experiences on involving teachers in research in Sweden.

Altogether, two main ideas emerged from the presentations and the subsequent discussion. The first is that we are indeed making progress, given the high number of successful experiences in which research and practice come together in small- and large-scale projects in productive ways. This concerns only specific cases and is not the general rule. But it is a starting point that requires analysis to widen our understanding of these successful projects and to become empowered to multiply them.

The second idea is that bridging the gap between research and practice, whether small- or large-scale, requires the involvement of those with political power at local and national levels. These may be subject department leaders, school leaders, regional leaders, or decision makers at ministries of education. Their involvement in validating guidelines and providing resources for transformative projects is critical, so that teachers and researchers participating in these processes can find appropriate working conditions and get recognition for their efforts in improving mathematics education.

The research-practice gap as an implementability problem

In the field of mathematics education, the divide between research findings and their practical implementation has long been a formidable challenge. The disconnection between what research suggests as effective pedagogical approaches and how these insights are integrated into classroom practices has raised significant concerns within the mathematics education community. This disconnection, often referred to as "the research-practice gap", underscores a critical issue that jeopardizes the efficacy of educational advancements and hinders the transformative potential of innovative mathematics teaching practices.

Drawing inspiration from Bruckheimer's seminal work in the late 1970s, where he delved into the implementation challenges of curricula across different educational systems, it becomes evident that the essence of this gap lies in the complex implementation process. Bruckheimer (1979) argued that implementing educational innovations cannot be perceived as a static and straightforward transfer of ideas from theory to practice. Instead, he proposed a dynamic perspective, wherein curricular implementation is viewed as an ongoing evolution. This evolution entails manipulation, adaptation, and additions to existing educational frameworks, emphasizing that effective implementations should inherently encompass creativity.

Central to this discourse is the notion of "implementation research," as outlined by Century and Cassata (2016). This research approach is characterized by its systematic inquiry into the enactment of innovations within controlled settings or everyday educational contexts. It seeks to unravel the multifaceted interplay between innovations, influential factors, and outcomes, offering a comprehensive understanding of the dynamics shaping successful implementation. By elucidating the factors that influence the translation of research into practice, implementation research serves as a bridge that narrows the research-practice gap (Jankvist et al., 2021).

This section of the paper argues that the research-practice gap can be fundamentally understood as an implementation problem. The complexity of this challenge necessitates a comprehensive exploration of innovative strategies that can enhance the alignment between research discoveries and classroom practices. Addressing the implementability problem could lead to transformative advancements in mathematics education, fostering a more harmonious integration of research and practice and propelling the field toward greater pedagogical effectiveness.

Why is it so difficult to make real progress on this issue?

The issue at stake in the relationship between research and practice is that mathematics education as a research field produces knowledge (theories, solid findings, didactical designs, etc.). However, this knowledge seldom has a foothold in practice. In other words, the accumulated knowledge from our field rarely reaches classrooms and school curricula, and it is utilized by end-users such as practitioners and policymakers. But why is it so difficult to make real progress on this issue?

Part of the difficulty lies in the fact that this issue has multiple layers of complexity. First, there is the problem of communicating research to practitioners. This problem is closely related to the concept of implementability of research-based innovations. This is, “how realistic and feasible it is for practitioners to implement an innovation arising from interpreted research results” (Jankvist et al., 2021, p. 1049). Communicating research findings in mathematics education to practitioners presents a significant challenge. The creators of theories often employ complex conceptual tools which, although necessary for the intricacies of the research, can act as a barrier to effective communication and implementation in practical settings. As noted by Koichu et al. (2021):

> Creators of theories might consider how to coin their concepts and constructs so that at least some of these would be communicable beyond the research community. Otherwise, the evident complexity of conceptual apparatuses used by mathematics education researchers may pose yet another obstacle for implementability of research. (p. 986)

However, even if we were to overcome the issue of communicating research findings to practitioners, other potential problems must be considered in the relationship between research and practice, and in the implementation of educational innovations based on mathematics education research.

One potential problem to consider is the *shift in ownership of an innovation*. Innovations are often developed by researchers who deeply understand the theoretical and conceptual underpinnings of the problem at hand. However, the end-users of these innovations, such as teachers, students, or educational institutions, may have different needs, constraints, and beliefs that the innovation needs to address fully. As the ownership of the innovation shifts from the developers to the end-users, there is a need for a thorough understanding and consideration of these factors to ensure the successful implementation and sustainability of the innovation. This may involve adapting the innovation to fit the context of the end-users better or providing additional support and training to help overcome potential barriers to implementation.

Other potential problems are the *fidelity and adaptation of an innovation*. This is the ability of the users to modify the innovation to fit their specific needs and constraints and the degree to which this modification preserves the original intentions and effects planned by the innovator. Practitioners may need to adapt innovative tools, methods, or curricula to their unique classroom settings, students’ needs, or institutional constraints. However, this adaptation may lead to a deviation from the original design and intent of the innovation. A key question here is, to what extent can the innovation be adapted without losing its intended benefits? The producers of research-based innovations need to provide clear guidelines on the core components of the innovation that must be maintained for it to be effective and the aspects that can be adapted to fit different contexts.

The problem of the *sustainability of an innovation* should also be considered. The sustainability of an innovation is a critical concern that involves understanding the necessary conditions required for the innovation to continue being effective over time, even after the initial implementation period. In the context of mathematics education research, this is particularly important because the ultimate goal is to create lasting improvements in teaching and learning processes. For an innovation to be sustainable, it must be integrated into the existing educational system and practices and have ongoing support from all stakeholders involved, including teachers, school leaders, students, and parents. Additionally, the innovation must demonstrate clear benefits that justify its continued use.

From our position as researchers in mathematics, it is also important to consider the issue of *making implementation an integral part of research*. This issue highlights the gap between research and practice in mathematics education. Researchers may design and test innovations meticulously, but implementing and adopting the innovation in real classrooms often needs to be followed up with the same rigour. To address this issue, it is essential to consider the implementation and adoption process as an integral part of the research. This task involves developing and testing the innovation, planning for its implementation in real-world settings, providing the necessary support and training for teachers, and continuously monitoring and evaluating its effectiveness in different contexts. As noted by Cai et al. (2017):

> If researchers hope to impact practice, they might need to embed implementation as an inseparable aspect of intervention design. If researchers leave implementation to teachers, or if they study implementation as a phenomenon separate from the design of an intervention (e.g., writing and testing curricula), they could miss a chance to boost the impact of their work. (p. 344)

In conclusion, bridging the gap between research and practice in mathematics education demands collaborative efforts from both researchers and practitioners. For research findings to genuinely impact educational practice, clear communication, adaptability, and sustainability are essential. While some progress has been made, the mathematics education community must persistently seek strategies to translate research into effective classroom practices, ensuring lasting benefits and improvements in mathematics teaching and learning.

Genetic constructivism in mathematics: Birth and spread of an innovative teaching approach in the Czech Republic

Implementing innovative mathematics teaching based on research results at a large scale is a great challenge and has become the focus of much attention (e.g., Koichu et al., 2021; Maaß et al., 2019). Such implementation needs to take into account the perspectives of practice, research, and policy. Krainer (2021) introduced the term *societal rationality* to describe an implementation of reform efforts standing between *technical rationality,* which builds on the view of externals looking at practice (top-down), and *reflective rationality,* which rests upon the view of the internal experts (bottom-up) (Schön, 1983). The birth and spread of a specific approach to teaching mathematics called genetic constructivism (GeCoT) in Czech schools fall within societal rationality.

The process started in the 1960s and 1970s under the communist regime, at the time of the Iron Curtain, when Milan Hejný, his father, and their collaborators (with virtually no links to international research at that time), based on observations of pupils' learning and struggling with mathematics in lessons, conducted the first studies on how pupils acquire mathematical knowledge. Viewed through today's lens, they would not meet the requirements we now have for research studies, nevertheless, they became the basis for a specific concept development theory, later called the theory of generic models (Hejný, 2012). Since the revolution in 1989, when international research became available, this theory has been linked to other theories and used extensively in Czech and Slovak research studies to account for pupils' mathematical learning.

Simultaneously, the theory of generic models became the foundation for a specific approach to teaching mathematics called genetic constructivism in mathematics teaching (Kvasz & Pilous, 2020).[1] In short, in GeCoT, pupils are at the centre – not only their learning but also the development of their personality, and motivation for and enjoyment from doing mathematics. Frontal teaching shifts towards pupils' active work with mathematics. Pupils solve tasks graded in difficulty and formulated within didactic environments.[2] They share solutions; collaboration among pupils is supported. Teachers are facilitators; they guide and mediate discussions with and among pupils. Errors are used as a springboard for learning.

Within about 20 years, GeCoT spread from a modest beginning to about 20% of Czech basic schools (elementary and lower secondary level) via an integrated network of stakeholders. We will briefly elaborate on the milestones and hurdles in this process.

In the 1990s, elements of GeCoT were included in the university education of pre- and in-service teachers and were implemented in classes via diploma theses supervised by Hejný and his team. Using the teachers' feedback, teaching materials based on the principles of GeCoT were gradually developed and trialled in schools with the help of a growing number of cooperating teachers, through the methodology of action research (see Zehetmeier's contribution below) and design-based research. In 2005, the Grade 1 mathematics textbook was published, and the textbooks for Grades 2 to 9 followed in due course. These textbooks differ vastly from the other mathematics textbooks in the Czech Republic. They do not include explanations of new knowledge followed by exercises to practise it, but they consist of didactic environments which permeate all the grades and in which pupils are asked to solve tasks graded in difficulty and, through their solutions, get to the new knowledge. The formulas and algorithms are not given as the ultimate goal of learning mathematics but understanding the process leading to them is given precedence.

[1] Due to the first publisher of Hejný et al.'s textbooks, it is locally known as Hejný method (https://www.h-mat.cz/en/principles).

[2] See, e.g., Hejný et al. (2015) or Jirotková and Slezáková (2013). Didactic environments may be likened to Wittmann's substantial learning environments (2001).

The textbooks found their way into schools quite slowly. It was difficult for most teachers who had taught in a traditional frontal way to shift toward constructivist ways, which place pupils in the centre. Some teachers originally enthusiastic about the new approach abandoned it as they used the textbooks in frontal transmission ways, which did not produce the changes in pupils' understanding and approaches which they desired. It became clear that there was a need for teacher training. An independent association H-mat was established with the support of sponsors to cater for the publication of textbooks, train quality lecturers and systematically nurture their professional growth. With the help of such certified lecturers, it was possible to train a growing number of teachers to implement GeCoT according to Hejný and his team's aims.[3]

However, there have also been other hurdles to overcome. Much effort has had to be put into the work with the general public, media and especially parents and school principals. Some parents could not understand how GeCoT worked and claimed that what is taught is not mathematics. Some were worried that their children would not know their multiplication tables. Hejný and his team took such worries seriously, and with the support of H-mat, they established so-called mathematics cafés in which certified teachers, the core team or practising teachers can work with parents and other interested parties and introduce them to the work in learning environments.

After the number of schools using GeCoT had grown and the method began to appear in the media, it drew the attention of some research mathematicians. While many were positive about the method as they felt pupils' understanding of mathematics was at its core and that is what they cared about most, there was a small number of mathematicians and other professionals who strongly opposed the method. They were vocal in the media and in 2016, tried to influence members of the parliament to ban the method in schools. They failed in their efforts, also thanks to the media and practising teachers. Those teachers who used GeCoT and were satisfied with it as it helped their pupils to understand and enjoy mathematics, resisted the pressure and stuck to the method.

To conclude, according to Krainer (2021), reform efforts gain in quality if all concerned (policy, research, and practice) are involved in the problem definition and in the solution and evaluation process. As can be seen from above, the new teaching approach originated at the university, was based on university research and spread mostly via its proponents and their cooperating teachers and student teachers. What about the third vertex of the triangle, policy? Schools and teachers have much autonomy in selecting textbooks and teaching and assessment methods in the Czech Republic (Hošpesová & Novotná, 2020). The GeCoT textbooks were approved by the Ministry of Education, Youth and Sports, which is standard practice in the Czech Republic, and while policymakers did not specifically support the new teaching approach (either financially or otherwise), they did not hinder the efforts of teachers and schools when implementing it, neither did they succumb to the attempts described above to ban the method. In response to the worries of

[3] Such education is ongoing; each year, 2,000-3,000 teachers participate in summer schools, seminars and lectures, many of them repeatedly.

the public and the above group of dissatisfied professionals, the Ministry commissioned a secondary analysis of TIMSS 2015 and 2019 data to determine how Grade 4 pupils taught by GeCoT achieved in comparison with others (Greger et al., 2022). The results were encouraging, further supporting teachers who advocated the method.

Teachers are key players in implementing new curricula and new ways of working (Vithal & Shimizu, 2023). The implementation of GeCoT is an example of the need to organise teacher education and professional development to make new teaching approaches sustainable. "Teacher involvement, ownership and commitment to the reform are regarded as important for successful implementation of a curriculum reform, and both bottom up and top-down strategies are needed" (Vithal & Shimizu, 2023, p. 558). If we change 'reform' into 'GeCoT', the statement remains true.

Action Research as a method to bridge the theory-practice gap

The theory-practice gap is a topic of central interest, from both research and practice perspectives. This gap refers to possible disparities between theoretical knowledge (in academic settings) and practical application (in real-world situations). It is crucial to bridge this gap to enable effective problem-solving, to allow the use of theoretical knowledge in real-world problems, and to foster efficient and productive processes. In particular, concerning the topic of professional development, bridging this gap is vital for personal and professional growth (see, e.g., Zehetmeier, 2015).

According to Rauch et al. (2019), action research was first conceived in the USA and later in the UK, Australia, and some European countries, including Austria, in the 1970s and 1980s. It was initially introduced by Lewin (1948) and was oriented to problem-solving in social and organizational settings; in this context, it showed parallels to Dewey's (1933) conception of learning from experience. Today it significantly impacts many professional contexts, particularly in teacher professional education (see, e.g., Rauch et al., 2019).

Action research can help bridge the theory-practice gap by combining rigorous research with practical application. It is a cyclical process that involves identifying a problem or issue, designing and implementing an intervention, collecting and analysing data, reflecting on the findings, and making changes based on the results. In particular, action research allows for a direct connection between theoretical knowledge and real-world practice. It emphasizes the integration of theory and practice by encouraging practitioners to critically examine their own actions and experiences and to use theoretical insights to inform their decision-making processes. Moreover, action research is conducted within specific contexts, such as educational institutions. By working directly within these contexts, researchers gain a deep understanding of the unique challenges and complexities involved. This kind of contextual understanding helps to bridge the gap between abstract theories and practical realities. In this way, action research also requires collaboration between researchers and practitioners. By actively involving practitioners in the research process, their practical knowledge and expertise are integrated into the research design, implementation, and analysis. This collaboration ensures that the research is relevant, practical, and directly applicable to the specific context. This process encourages practitioners to engage in reflection and self-evaluation, since the iterative cycles of planning, acting, observing, and reflecting, allow them to critically evaluate their own assumptions, biases, and practices. By actively engaging in this cyclical research process, practitioners can adapt and refine their practices based on the feedback and insights gained from each cycle. This iterative approach helps to bridge the theory-practice gap by fostering a dynamic and evolving relationship between theory and practice. This reflective practice also allows a deeper understanding of the theory-practice relationship and supports the integration of new insights and knowledge into daily practice (see, e.g., Altrichter et al., 2014; Feldman et al., 2018).

One example of this is the research topic “teachers’ learning”: Here, effective ways of promoting teachers’ professional development are of great interest, in particular for both the participating teachers and the researchers. From a holistic perspective (according to Pestalozzi’s idea of learning by head, heart, and hand), the major indicators for describing teachers’ learning are their knowledge, beliefs, and practice (Zehetmeier, 2015). However, the situation is rather complex since each of these notions can be defined in different ways. Teachers’ knowledge, for example, can be differentiated into content knowledge, pedagogical knowledge, and pedagogical content knowledge; but it can also be regarded as knowledge about learning and teaching processes, assessment, evaluation methods, and classroom management. Similarly, teachers’ beliefs can include different aspects of beliefs about particular subjects and their respective teaching and learning. They also include the participating teachers’ perceived professional growth and their satisfaction, their perceived efficacy, and the teachers’ opinions, values, attitudes, and interests. At the practice level, teachers’ focus is on classroom activities and structures, teaching and learning strategies, methods, or contents. This example illustrates that knowledge about teachers’ learning comprises a wide range of possible aspects and dimensions, which – in sum – is rather complex.

At this point, a crucial question arises: Who is producing knowledge within the complex research topic of teachers’ learning? Krainer (2011) discusses this question of “knowledge production” (p. 50) and highlights two possible perspectives on this issue: Technical Rationality and Reflexive Rationality. According to Schön (1983), the term Technical Rationality is based on three core assumptions: There are general solutions to practical problems. These solutions can be developed outside practical situations (e.g., in research). The solutions can be translated into practitioners’ actions using, for example, publications, training, or administrative orders. In contrast, Reflective Rationality is based on different assumptions (Posch, 1996): Practical problems require particular solutions. These solutions can only be developed inside the context in which the problem arises and in which the practitioner is a crucial and determining element. The solutions cannot always be applied to other contexts, but they can be made accessible as hypotheses to be tested in practice.

Thus, Reflective Rationality puts teachers’ expertise into focus. The concept of action research is closely connected with this perspective since it uses teachers’ own actions and experiences as the starting point for both their professional development and research. This attributes to teachers (and their perspectives) a central role in the process of knowledge production. Thus, in sum, action research can offer a complementary perspective for research.

According to Krainer (2011), there is a rather long tradition to viewing teachers as experts and research partners. For example, they have been described as researchers (e.g., Stenhouse, 1975), reflective practitioners (e.g., Schön, 1983), or experts (e.g., Bromme, 1992). Together with researchers from academia, teachers can generate knowledge regarding core questions of educational research: "The ideal way would be to regard researchers as key stakeholders in practice, and teachers as key stakeholders in research" (Krainer, 2011, p. 59).

To conclude, we share some personal thoughts and experiences from previous CERME conferences, regarding our particular contexts and ourselves as researchers: in different research contexts, we use different notions and frameworks (e.g., regarding teachers' knowledge/beliefs/practice), we assign teachers different roles in our research (e.g., research on, of, with, and for teachers), and we have different perspectives on how to bridge the theory-practice gap. Thus, there is an urgent need to make clear our particular contexts, why we use our notions and frameworks, and how they relate to other notions and frameworks. As researchers, being open to other perspectives can help us to challenge our own assumptions (that we might take for granted), e.g., how mathematics should be taught, how teachers should be educated, or how research should be done. These discussions can help us to shift those assumptions by working across boundaries, thinking about implications for our own research, doing this research in responsible and inspirational ways, and bridging the theory-practice gap.

Lesson Study: Research-informed professional development for teachers

Mathematics education in the UK is in crisis. There is a severe shortage of mathematics teachers, meaning almost half of secondary school mathematics teachers are non-specialists (Times Education Supplement, 2022). UK primary teachers do not specialise and many have weak subject knowledge (Witt et al., 2013). Add to this the fact that the UK has some of the highest levels of maths anxiety amongst students and you have a recipe for a mathematics crisis.

How can academics support those teaching mathematics to access the wealth of high quality research UK mathematics education departments produce, so that it impacts on students in UK classrooms? Over the last few decades in the UK, there have been numerous calls from policy makers, academics and practitioners for there to be a closer relationship between research and practice (e.g. Coe et al., 2015). As a result, organisations like the Education Endowment Foundation and Cambridge Maths have worked to make research material more accessible and digestible to teachers through a series of easy to read practice guides and research summaries, all freely available online for teachers. So the UK has overcome the challenge of research material being dense and inaccessible but is still struggling to connect teachers to research in a way that improves practice.

What is it that will encourage teachers to engage with mathematics research material and, more importantly, to apply it to their practice? In this contribution I will explore lesson study as one solution to this challenge, and the practical implications of its implementation in schools, drawing out lessons for mathematics professional development more broadly.

Godfrey (2017) distinguishes between three approaches to teacher engagement with research:

- 'evidence-based practice', a passive process in which teaching approaches are based on evidence about 'what works' produced by academics, predicated on the belief that teachers cannot be researchers, or that their research is of an inferior quality;
- 'evidence-informed practice' places an equal value on teacher generated research findings, and teachers actively combine evidence from academic research, practitioner enquiry and other school-level data;
- 'research-informed practice', when teachers engage in and with academic and practitioner forms of research, using evidence from both to make changes to practice.

For Godfrey (2017), it is 'research-informed practice' which has the biggest impact on practice, giving teachers agency both to explore what works for them in their context and to generate new learning for other teachers to use.

On Lesson Study

Lesson study is one example of 'research-informed practice', born in Japan but widely used internationally, which enables teachers of mathematics to engage in structured cycles of 'research-informed practice'.

Starting by identifying a research question, lesson study has several key process components that enable teachers to connect research to practice. Firstly, teachers collaboratively plan a lesson to explore their research question. This stage involves a process called *kyozai kenkyuu*, a difficult term to translate into English, but which involves teachers in a study of research material relevant to the research question, and which will inform the lesson plan (Murata, 2011). In Japan, this material includes evidence from other schools' lesson study cycles, usually published in Japanese bookshops (Fujii, 2016), so it is highly relevant to 'real' teachers and their own mathematics classrooms.

Secondly, the lesson plan enables research findings to be incorporated and tested in live practice through the research lesson. Using the collaboratively written plan, one teacher teaches the lesson, whilst others gather evidence as to its success in relation to the research question (Seleznyov, 2019). This aligns with a large body of research around the need for teachers to be enabled to actively test out new learning in the classroom if professional development is to be effective (e.g., Darling-Hammond et al., 2017).

Once the research lesson has taken place, evidence gathered by teachers observing the lesson is shared and discussed in a formal post-lesson discussion (Saito, 2012). Again, the research teachers have studied is referred to in this discussion as they explore the extent to which the theories they borrowed from research were enacted in the classroom. Stoll (2015) describes how powerful collaborative conversations around shared practice build trust, encourage non-judgemental challenge and increase the likelihood of practice change.

An ‘outside expert’, known as koshi supports both planning and research lesson stages, offering expert advice that will enable the group to connect the research to their practice (Fujii, 2016). The koshi is a lesson study and subject expert, often from an academic background (Fernandez & Yoshida, 2012), who operates across networks of local schools: again, a bridge between research and practice. Through koshi networks, publication of findings and ‘open house’ research lessons, in which visitors across the district attend and observe research lessons and post-lesson discussions (Fernandez & Yoshida, 2012), lesson study offers Japanese teachers a system-wide approach to ‘research informed practice’.

Challenges to the UK implementation of Lesson Study

If lesson study offers such a neat solution to the gap between research and practice for mathematics teachers in the UK, why has it not been more widely adopted in UK schools?

For most UK teachers, observations of practice are to be feared (Wood, 2016). UK schools have spent decades using lesson observation as a tool to judge and punish teachers in the UK, meaning teachers usually frame lesson observation as a means of performance and judgement, instead of learning and improving (Watkins, 2010). This is particularly acute for mathematics teachers with weak subject knowledge. The collaborative ownership of the lesson plan in lesson study reframes this process as one of joint ownership and collective endeavour, reducing the possibility of negative judgement,

Another challenge to the translation of lesson study is the focus on short-term measurable impact in the UK. Lewis et al. (2006) compare the US response to lesson study, with researchers “proposing randomized controlled trials and horse-race style comparisons’ to Japan, where lesson study ‘has been used for a century without summative evaluation” (p. 6). In the UK, there is similar pressure on schools to prove impact through short-term pupil outcomes and to demonstrate value-for-money. Japanese lesson study focuses on the development of expertise over decades, not months (Lewis et al., 2006), both at individual, school and system level. Time enables teachers to surface theories of learning and ensures greater depth of learning (Cordingley et al., 2016).

Finally, opportunities for knowledge mobilisation are limited in the UK system where lesson study is not an embedded system of teacher learning. We have no koshis, and those ‘outside experts’ schools could access would require payment for their time, something schools cannot afford. In addition, teacher workload is a national issue, meaning teachers are reluctant to take on work beyond their contracted hours, unlike in Japan, where much lesson planning is done in teachers’ own time (Fujii, 2016).

Making Lesson Study work in the UK

Despite these challenges, there are some UK schools using lesson study as a whole school professional development approach, and it is supporting teachers to both bridge the research and practice divide, and to improve teaching and learning. Seleznyov (2023), studied three case studies of UK schools using lesson study to explore the factors that enabled lesson study to become a sustainable model of research informed practice. In these schools, the role of the 'Lesson Study Champion' is key.

The school leaders who acted as Lesson Study Champions in the schools studied were highly knowledgeable about the practice being implemented, and would problem solve and support those implementing the change, persevering even when implementation became more challenging (Aarons et al., 2014; Moullin et al., 2018). Champions were passionate about the implementation of lesson study, overcoming any indifference and resistance, 'going the extra mile' by dedicating extra time and energy to the implementation of lesson study. They did not always realise their own importance in terms of the practice they were seeking to implement, aligning with Moullin et al.'s description of them as 'humble leaders' (2018, p. 5); to demonstrate this, they acted as participants in the process, as well as leading it for others.

The Champions took several concrete actions to ensure lesson study became a sustainable model of professional development. They redeployed professional development time and funding to make lesson study fit within teachers' working hours, and did this consistently over time. Where certain stages of the process proved culturally alien or tricky to implement, they found local adaptations that worked whilst trying to remain true to the original principles of Japanese lesson study. For example, it was challenging to share knowledge across teacher groups, so the schools tried sharing events, publishing findings and having facilitators working across several groups. They made careful succession plans, training others in lesson study and imparting the same passion and persistence to them, to ensure that lesson study would have longevity even if the Champion left the school. As a result, teachers interviewed for the study all agreed that lesson study had become a part of 'the way we do things' in their school and would remain even if senior leaders moved on.

Conclusion from the Lesson Study case

Lesson study is one example of a professional development process that connects research and practice for mathematics teachers. What are the implications of this example for those seeking to bridge the research and practice divide for in-service teachers? Firstly, professional development process should seek to enable teachers to immediately incorporate research findings into practice: live teaching is the ideal vehicle for this and repeated cycles of study, live teaching and reflection, which lesson study incorporates, create a system in which research is continually tested out in the classroom. Secondly, in order to implement professional development systems in schools, leaders must reorganise professional development time and commit resource to enable sustainability, advocating for research-informed practice with passion and persistence, problem solving when implementation problems arise and being a participant to demonstrate their commitment to the approach.

Involving teachers in research

In this panel, we focus on practice-based research. For such research, the involvement of schools, teachers and their pupils is crucial. In this contribution, one example of involvement will serve to illustrate what a fruitful collaboration between teachers and researchers can look like and how such collaboration can evolve over time (see Palmér & van Bommel, 2021). The goal might not need to be teachers becoming researchers, but teachers and researchers knowing and valuing each other's strengths and weaknesses: a collaborative understanding of what drives research and what drives teaching. Gaining such understanding means that the collaboration (partnership) between researchers and teachers becomes more productive and sustainable (Cai et al., 2019).

Within the context of early childhood education (preschool class, 6-year-olds, Sweden), two researchers started by testing different tasks in different preschool classes. They ‘borrowed’ pupils from preschool class teachers to test and validate their designed tasks. They then redesigned the tasks after analysis of collected data and where possible, tested again with new pupils in new classes. The role of the researchers was to design, implement, analyse and re-design tasks. The role of the teachers was more or less undefined. Most teachers chose to observe their class while the researchers were acting as teachers (Figure 1).

Once the tasks were constructed and tested, teachers were involved in other ways. At first, a group of teachers received instructions on how to implement the tasks in class, and took part in discussions before and after implementation. They started implementing the designed tasks in their own classes and reflected on the outcome together with the researchers. The teachers implemented and reflected, but the design, analysis and re-design was still done by the researchers.

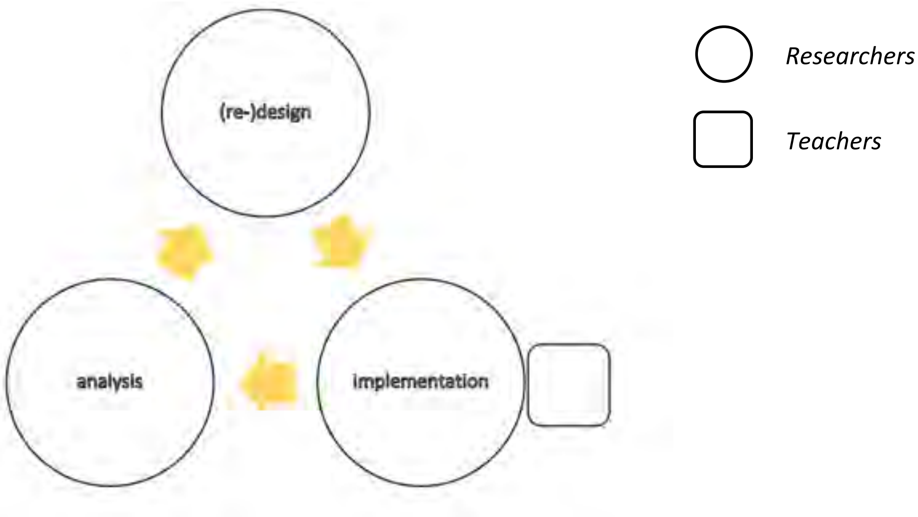

Figure 1: The role of teachers and researchers in a design cycle (first phase)

Some of the teachers showed an interest in further development and collaboration and after some years of collaboration, their involvement was formalized and systematized, partly by allocating hours to the project, but also by including the teachers in the whole design cycle. This resulted in a situation where teachers not only implemented tasks in class, but were involved in the design of suggested tasks and the analysis (Figure 2). Teachers now reflected on pupils' actions and results, which in turn positively influenced teachers' role in the design and re-design of problem tasks and implementation. Researchers were not involved in the implementation, but on occasion, they observed the implementation in order to generate a better understanding of the challenges in specific classrooms. Thus the traditional roles of teacher and researcher are redefined (Cai et al., 2018).

When teachers are involved in this way, new ethical issues arise (see Palmér & van Bommel, 2021). Can the data be considered authentic when the teaching is done by teachers? Through recurring discussions, researchers and teachers try to agree on the goal of the implementation and the data collection. Detailed teaching instructions are jointly discussed and developed, aiming to obtain comparable interventions but taking the specific classroom into consideration. The interesting part here is the authenticity of the implementation. One of the teachers says:

> They know us, they are safe with us. If someone new comes... of course it can work too, but I can imagine that there will be different results. Because they [the pupils] know us and they are safe with us, and then nothing will be strange. It's a bit like a regular lesson. So, the result will be as it should be, as in real life.

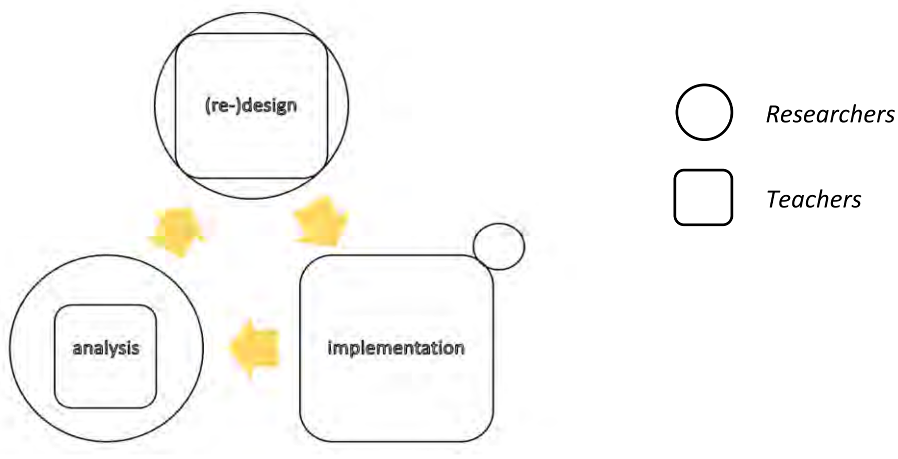

Figure 2: The role of teachers and researchers in a design cycle (current phase)

This reflection allows us to put the pupils and their experience in the spotlight. Instead of the researcher implementing and taking ethical aspects into consideration while teaching, this way of collaboration offers an alternative. When the teacher is the one implementing, the data and thus the results might be more authentic.

How to write for teachers and how to make them read?

Maybe we should ask ourselves the question, what should we write for teachers, so it is worth their time to read? The gap between research and practice is often mentioned, as well as the issue of translation: how to translate research results into practice (see points made by Vondrová, Zehetmeier and Seleznyov earlier in this panel-contribution). When working together with teachers in all parts of the design cycle and throughout the whole design cycle, a translation is not needed to the same extent. The translation has already taken place before starting the implementation, it is in the design and permeates all planning and discussions between teachers and researchers in the project. The research questions are formulated from the perspective of the practice, bridging the gap. Subsequently, the results, as the teacher above says, emerging from 'real life'.

The Swedish government showed their awareness for this research-practice gap and acknowledged the need for collaboration between practice and research. When the Association of Swedish Higher Education Institutions suggested the so-called ULF-agreement (education-learning-research) aiming to explore different forms of sustainable collaboration models between academia and schools, the Swedish government supported this initiative (Regeringsbeslut U2017/01129/UH). In research projects within this agreement, teachers and researchers collaborate. In the long run, these types of collaboration should decrease the perceived gap and increase the scientific grounds of teaching. An external evaluation of the initial phase (2017-2021) showed that ULF strengthened practical research in the field of education and offered opportunities to bridge the gap through teacher-researcher collaboration (Prøitz et al., 2022).

What other forms of communication may be effective in reaching teachers?

As researchers we might need to ask ourselves the question, where do teachers go to get their information, and where can we publish so our results are more accessible to teachers? This might be traditional scientific publications or conferences, but it might also include newer ways of dissemination, like different forms of social media where teachers manage their own professional development (van Bommel et al., 2020).

Dissemination might also happen if we rephrase the question above: 'how to write for teachers' into 'how to write with teachers'. In the case described above, the participating teachers presented at (teacher) conferences and were thus involved in the dissemination of research. They chose the content and form of their communication with other teachers. When writing and presenting together, or when supporting teachers with presentations and publications of all forms, content will be adapted for and by teachers, and thus, bridge the gap.

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