RELATING MATHEMATICS AND LITERATURE AS A TEACHING STRATEGY AT THE HIGH-SCHOOL LEVEL

Lina Medina Ibarra¹ · Avenilde Romo-Vázquez¹ · Mario Sánchez Aguilar¹

Abstract
We report the application of a didactic design that relates literature and mathematics and is embedded in the paradigm called «questioning the world» as proposed by Chevallard (2013). The design was implemented in two groups from a high school located in a marginalized urban area of Mexico. The heart of this activity is the analysis, from a literary as well as from a mathematical point of view, of the story of Jorge Luis Borges “The library of Babel”. Some of the elements analysed by the students were the form of the Library of Babel, and the number of galleries, books and pages contained in it, which in turn motivated the articulation of ideas about the notion of infinity. Besides pretending that students explore the mathematical ideas involved in the story, the aim of the design is also to promote a critical thinking among students and provide them with tools to question their reality.

Introduction
The use of literature into the teaching of mathematics is not a new concept, different scholars have argued about the benefits of integrating these two disciplines. For example, through the use of literature in the mathematics classroom students’ interest in literature and mathematics can be fostered, but such integration also facilitates the introduction of new mathematical ideas in the classroom, it encourages the development of critical thinking, and promotes the understanding of new mathematical concepts. In this manuscript we report a teaching experience at the high school level that is based on the integration of literature in the teaching of mathematics.

The teaching experience consisted on the application of a didactic design called “A library for your community” that had a dual purpose: on the one hand, to

¹ Instituto Politécnico Nacional, CICATA Legaria, Programa de Matemática Educativa, Mexico
✉ lmedinai1400@alumno.ipn.mx · aromov@ipn.mx · mosanchez@ipn.mx
engage students in the analysis of a literary work that contains mathematical notions, and to promote the understanding of such notions; on the other hand, to foster students’ reflections on the characteristics and role of libraries in their community.

To place our work in the landscape of research on the use of literature in the teaching of mathematics, we present a brief review of this area of research. Later we clarify the theoretical position in which our experience is framed. We then detail the method followed for the implementation of the teaching experience. Finally we show some of the results of the implementation, and we mention some conclusions.

**Use of literature in the teaching of mathematics: a brief review**

For at least 25 years, there has been an interest on the part of mathematical educators from different regions of the world to use literature as a means for the teaching of mathematics (see for example Welchman-Tischler, 1992). Although most research on the use of literature in mathematics teaching has been developed at the preschool and elementary school levels (see Flevares & Schiff, 2014, for a comprehensive review on teaching and learning early childhood mathematics with children’s literature), there are some reports of teaching experiments at the high school level (for example Sriraman, 2003; 2004).

Researchers have reported different benefits of integrating literature into mathematics teaching. Such benefits can be classified into two categories: learning benefits and affective benefits.

**Learning benefits of using literature to teach mathematics**

One of the benefits that have been identified is that the use of literature in the teaching of mathematics can promote students’ problem solving and problem posing abilities. For instance, Young & Marroquin (2006) illustrate how mathematics teachers can pose rich mathematics problems through their contextualization in scenarios and stories from children’s literature. These researchers conclude that such literature-based problems not only enhance problem-solving skills, but also encourage communication and justification processes.

Another benefit reported in the literature is the development of students’ critical thinking. An example of this is the work by Sriraman (2013), who asserts that critical thinking is a common element between literature and mathematics. This author organized a teaching experiment of several sessions in which 13-14 year-old students read and analysed the contents of Edwin Abbott’s Flatland book. Sriraman (2013) argues that this activity allowed students to develop their critical
thinking, and to be introduced to sophisticated mathematical ideas such as non-Euclidean geometries.

It has also been suggested that books have the potential to encourage students to make connections between mathematical ideas and their experiences outside of school. For example, Hellwig et al. (2000) illustrate how trade books (this is, books other than textbooks, in mathematics instruction) can help students to connect mathematics with other content areas, and to identify connections between different ideas in mathematics.

**Affective benefits of using literature to teach mathematics**

Another merit of the use of literature as a means to teach mathematics is that it can produce emotional and motivational gains in students. An example of this is reported in the study by van den Heuvel-Panhuizen & van den Boogaard (2008) where they read pictures books to young children to stimulate mathematical thinking about geometry, data representation and measurement; however, these researchers claim that these books also have the power to engage and focus the attention of a group of children, which in turn can facilitate interactions between the children. Similarly, Hong (1996) analyse the effectiveness of using children’s literature to promote mathematics learning, particularly the effects on dispositions toward mathematics learning, concluding, “the disposition to voluntarily pursue mathematics learning can be increased using children’s literature” (p. 488).

The use of literature in the teaching of mathematics also has the potential to support the mathematical learning of students with different backgrounds and abilities. Iliev & D’Angelo (2014) assert that the inclusion of multicultural literature in mathematics classroom “enables children to move beyond their current knowledge base and make culturally relevant mathematical connections through such universal topics as money, time, and measurement” (p. 454). In turn, Courtade et al. (2013) claim that shared story reading, defined as: “when a teacher orally reads a book, and students purposefully and strategically interact with both the content of the book and the teacher” (p. 35), can be an effective mean to support the mathematical learning of students with moderate and severe disabilities. Moreover, Casey et al. (2008) suggest, “integrating literacy and mathematics through oral storytelling can enable low-income, diverse learners to acquire better geometry skills” (p. 43).

The aim of the teacher experience that we report in this paper was to generate both, learning and affective benefits for the students. On the one hand it was tried that the students analysed and deepened in mathematical notions such as infinity and the properties of some geometric forms; but on the other hand, we also
sought to promote students’ critical thinking, particularly in connection to the role that libraries play in their own community. Finally, we also wanted to foster a perspective among the students where literature and mathematics are not seen as two independent and unconnected subjects.

**Theoretical framework**

The Anthropological Theory of the Didactic (ATD) (Chevallard, 2013) frames the teaching experience reported here. In particular we use the notion of «Study and Research Paths» (SRP) which we explain below.

**Study and research paths**

Study and Research Paths (SRP) are a didactic device that emerges from the educational paradigm known as “Questioning the world”, in which problematic issues Q are studied, thus emulating the scientific activity, in which open questions motivate knowledge construction and the production of “acceptable” but partial answers.

The inquiry conducted by x on Q opens a route called study and research path. In order to move forward through this path, the inquiry team X has to use the knowledge —connected to the answers R, as well as to the works O— hitherto unknown to its members, with which the team must familiarize itself so that it can continue through the path towards the answer R (Chevallard, 2013, p. 170, our translation)

In this way, the Q issues enable the construction of knowledge. The study of Q must be done by a team of students X, where x is a particular student, oriented by a team of professors Y, or a teacher y, emerging thus a didactic system with the form (X; Y; Q). The operation of this system consists of searching for the answer to Q, through researching the existing answers R (knowledge) on O works (books, educational videos, webpages, etc.) that x evaluates and that allow her to generate sub-questions to guide her research. In order to move forward through this path, the inquiry team X has to use the knowledge about both, the R answers and the O works.

In sum, a SRP is a didactic device that organizes knowledge in a succession of pairs of questions and answers, where the questions can be related to mathematics and other disciplines. The relevance to the questions is fundamental to overcome the classic paradigm of “visiting the knowledge”, and introduces the new paradigm of “questioning the world”.
Method
The teaching experience reported in this manuscript is part of an ongoing research study developed primarily by Lina (the first author of this paper) as part of her master studies in mathematics education. Lina is a mathematics teacher in a Mexican high school, while Avenilde and Mario (second and third authors respectively) are her supervisors. It is important to note that during the planning and design phase of the SRP we also had the collaboration of Claudia, a literature teacher who works in the same school as Lina.

Selection of a literary work
First of all, a literary work was selected as a central element of the SRP. Lina and Claudia made the selection, and the selected work was “The Library of Babel” by Jorge Luis Borges. There were several reasons for selecting “The Library of Babel”: first, the story contains mathematical notions relevant to the students’ mathematical education such as the notion of infinity, and some geometric shapes such as hexagons; second, the story can not be understood in its entirety without a certain understanding of these mathematical notions; and third, Borges is one of the Latin American authors that students should read as part of their high school studies.

“The Library of Babel” is a short story where Jorge Luis Borges conceptualizes the universe as an immense library composed of hexagonal galleries. Figure 1 shows an extract from the English version of “The Library of Babel”.

The universe (which others call the Library) is composed of an indefinite and perhaps infinite number of hexagonal galleries, with vast air shafts between, surrounded by very low railings. From any of the hexagons one can see, interminably, the upper and lower floors. The distribution of the galleries is invariable. Twenty shelves, five long shelves per side, cover all the sides except two; their height, which is the distance from floor to ceiling, scarcely exceeds that of a normal bookcase. One of the free sides leads to a narrow hallway which opens onto another gallery, identical to the first and to all the rest. To the left and right of the hallway there are two very small closets. In the first, one may sleep standing up; in the other, satisfy one's fecal necessities. Also through here passes a spiral stairway, which sinks abysmally and soars upwards to remote distances. In the hallway there is a mirror which faithfully duplicates all appearances. Men usually infer from this mirror that the Library is not infinite (if it were, why this illusory duplication?); I prefer to dream that its polished surfaces represent and promise the infinite ... Light is provided by some spherical fruit which bear the name of lamps. There are two, transversally placed, in each hexagon. The light they emit is insufficient, incessant.

Figure 1: An extract from the English version of “The Library of Babel”

An English version of the text can be found at:
https://libraryofbabel.info/Borges/libraryofbabel.pdf
This short story has been analysed and illustrated by literary people, mathematicians, and people who is just interested in these disciplines (see for example http://libraryofbabel.info/). This makes available several Works O that, theoretically speaking, the students could draw on to develop the SPR “A library for your community”.

After the selection of the text, the design process of the SPR continued. The final version of the Study and Research Path consisted of three stages: (1) exploring and analysing the text; (2) towards the design of a library; (3) designing a library. Next, we explain each of these stages in more detail, but before that we will describe the characteristics of the context where the SPR was applied, and the characteristics of the participating population.

Context and population sample

The SPR was implemented in a private high school located in a marginalized urban area of Mexico. The implementation was carried out in two groups of high school students from fourth semester (16-17 years), and required four sessions of fifty minutes each one. The groups were organized into teams of between four and five members. In relation to the academic background of the students, at the time of conducting the experimentation they had studied the mathematical subjects of: algebra, geometry, analytical geometry and functions. In the case of subjects related to literature and language, they had taken courses in: reading and writing (two workshops) and literature (two courses).

Stage 1 — Exploring and analysing the text

The purpose of this first stage was for the students to analyse the structure of the library of Babel. For this, a set of triggering questions was posed to the students. Some examples of such questions are:

- What shape does a gallery have?
- Based on your calculations, is the library of Babel infinite or finite?
- What is the difference between an exaggeratedly large amount and infinity?
- Do you think that the library of Babel described in the story could exist. Why? Argue your answer.

Stage 2 — Towards the design of a library

At this stage students were asked to analyse elements for the design of a library in their own community, such as the type of terrain where it could be built, the budget to construct it, the type of design that could be proposed (its geometric
form), the books that could contain, etc. The aim was for a modelling activity to take place, and beyond the ideas of infinity and the literary aspects of the story, to analyse the role of a library in their community and how a project could be proposed to a governmental instance so that it could be materialized. Here it is important to remember that the school where the SPR was implemented is located in a community with few libraries. Some of the questions proposed for this phase are:

- Why are libraries important?
- How many libraries are in your community?
- Are they appropriate to meet the academic needs of the students in your community?
- What characteristics do you consider a library should have in order to meet the needs of your community?

Stage 3 — Designing a library

The aim of the third stage was for the students to build a model of a library for their community. This implies the use of mathematical modelling, to work on forms, scales, geometric design, but also involves analysing their social reality and how it could be modified. We think that through the library mock-ups developed by the students it could be possible to perceive how they imagine and propose a design of a library for their community.

Some results of the implementation of the SRP

In this section of the manuscript we will present some of the results obtained when implementing the SRP. In particular, we present some ideas related to the notion of infinity that the students manifested during the development of the SRP; due to space constraints it was not possible to present in this manuscript results related to the designs of the libraries developed and proposed by the students.

Students’ ideas on the notion of infinity

Several ideas on the notion of infinity appeared in the written reports delivered by the teams of students. One of them is that “too big is infinite”. For example, in figure 2 the team number 1 indicates that the library is infinite, but has a finite number of books (answer to question 3). They calculate the number of shelves and books, but assume that since the library is too big, then it is infinite. To argue that its size is infinite, they claim that this is because it resembles knowledge, which they conceive as infinity.
In a similar way, team 2 calculated the number of books, but they claim that it is not possible to calculate the number of galleries; in fact, they represent a gallery as an open structure (see figure 3). Thus, the fact of not being able to calculate the number of galleries makes them assume that the library is infinite, “the countless is infinite” they say.
Figure 3: Written report of the team 2, where they express some conceptions on the notion of infinity such as “the countless is infinite”

Another team of students (see Figure 4) noted that the library “is infinite because there is no limit”.

Figure 4: Written report of another team of students where they claim that the library is infinite because it has no limit

Conclusion

The SRP allows in the first instance to analyse the short story “The Library of Babel” considering both, literary elements (fiction and metaphor) as well as
mathematical elements (the geometric form of the library, its size, the number of
galleries, books and pages). This analysis forces us to question the main
metaphor in which the universe is viewed as a library, which makes us assume
that the too large is infinite. The questions proposed in the first stage of the SRP
triggered students’ critical thinking in the sense that they questioned the
information provided in the short story: can the library actually exists or just
corresponds to a fiction? The SRP is a didactic experience that questions reality,
on the one hand the reality portrayed in the story, and on the other hand
students’ reality: is it possible to propose a library for the community where I
live? How could I propose it? What would be the benefits? etc. Thus, students
are responsible for analysing the story and looking at their context as active
citizens that can question, analyse and propose projects for their community.

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