We present a literature review of the emerging research area online mathematics teacher education (OMTE). The review focuses on identifying (1) the main issues investigated in the area, (2) the main theoretical approaches employed, (3) the kind of empirical evidence that the researchers produce and present in order to support their findings and the way they analyze the data. Finally we use the concept of humans-with-media (Borba & Villarreal, 2005) to reflect on the possible transformations that online environments produce in the production of research knowledge within this research area. Our study provides an updated overview of the OMTE research area.

INTRODUCTION

In his book chapter Schoenfeld (1999) states that the research methods used in educational research are constantly evolving and they may even become obsolete:

To put it starkly, yesterday’s tools, techniques, and perspectives are valuable, but they are inadequate to cope with today’s challenges, just as today’s tools, techniques, and methods will be inadequate in just a few years. (p. 171)

If we look at the evolution of the field mathematics education research, we can see that, indeed, the research methods used in the field have changed (Hart, Smith, Swars, & Smith, 2009) and some of these changes are related to the emergence of new technological tools. For instance, today is possible to use software to develop detailed and comprehensive analysis of qualitative data such as gestures, speech and rhythm (Radford, Bardini & Sabena, 2007), or to study students’ conceptions of mathematics through analyses of the photographs taken by mathematics students (Harkness & Stallworth, 2013).

The initial motivation of this work was to explore how technological tools are transforming the work of contemporary researchers in mathematics education. We studied this transformation within an area of research in mathematics education that, by its very nature, takes place in technologized environments. We refer to the research area of online mathematics teacher education [OMTE] (Borba & Llinares, 2012). To explore the possible changes that technological tools produce in researchers’ work we use the concept of humans-with-media. This theoretical concept helps to explain how technological tools—and also non-technological media—influence and reorganize the way humans know and produce knowledge (Borba & Villarreal, 2005). Thus, in this paper we analyze, through the concept of humans-with-media, the activity of
researchers working in the area of OMTE; more particularly, we pay attention to the possible transformations that online environments produce in the production of research knowledge.

The meta-study reported in this manuscript is not limited to analyzing possible changes in the work of researchers brought by technological tools; our study also provides a characterization of the emerging research area OMTE. Thus, the purpose of this paper is twofold: (i) to provide a characterization of OMTE research area that focuses on the main topics studied and the theoretical and methodological tools used, (ii) to analyze the research knowledge produced by researchers studying OMTE, paying special attention to the role that technology plays in the production of such knowledge. Our study provides an updated overview of the OMTE research area that can function as a benchmark for future comparisons that could allow us to assess how this research area has evolved.

RESEARCH QUESTIONS AND METHOD

For practical reasons, to analyze the researchers’ work we didn’t make direct observations of their activity. We opted instead to analyze empirical studies on OMTE, recently published in international research journals. Our analysis of such empirical studies focused on identifying the main topics studied, the theoretical and methodological tools used, and the type of empirical evidence produced and presented in the manuscripts to support the findings. In particular, the research questions that we addressed in this study are:

- RQ1. What are the main issues investigated in the area of online mathematics teacher education?
- RQ2. What are the main theoretical approaches employed in this area?
- RQ3. What kind of empirical evidence do the researchers produce and present in order to support their findings and claims and how do they analyze the data?
- RQ4. Is the work of the researcher transformed by the characteristics of the online environments? If yes, how?

Empirical studies reviewed

The literature consulted to develop this study was divided into primary and secondary sources. Next we describe each of these categories.

Primary source

The area of OMTE has been characterized as an emerging research area, on which little has been published (Borba & Llinares, 2012). Borba and Llinares (2012) state that a literature search related to e-learning in mathematics education and OMTE in some of the major international journals during the period 2005-2012, produce just a few results. Aware of the scarcity of specialized literature, we decided to start our search in the special issue of the journal ZDM—The International Journal of Mathematics
Education devoted to the topic of OMTE (volume 44, issue 6) and which brings together researchers from different regions of the world which use different theoretical and methodological approaches in their studies.

Secondary sources
The secondary sources consulted for this study have different origins. On the one hand, we searched on the bibliographic references used in the articles obtained from the primary source. In addition, we consulted three international journals whose aims and scope are directly related to two constitutive elements of the OMTE: mathematics teacher education, and the use of the computers, Internet or other technological resources. The three journals consulted were the Journal of Mathematics Teacher Education; Technology, Knowledge and Learning (formerly known as International Journal of Computers for Mathematical Learning); and The International Journal for Technology in Mathematics Education.

It is important to note that all the manuscripts that were selected from the primary and secondary sources for further analysis met the selection criteria described in the next section.

Selection criteria for manuscripts
The manuscripts that were selected for further analysis had to meet the following conditions. First, the manuscripts should report an empirical study in the area of OMTE, these sorts of manuscripts would provide us with relevant information to answer the research questions, particularly RQ3. Second, the articles should have been published recently, more particularly, should have been published during the period 2009-2013. This last requirement allowed us to locate manuscripts that provided us with an updated overview of the state of development of the OMTE.

The above-mentioned selection criteria were applied to the primary and secondary sources for selecting the manuscripts; for example, we selected eight manuscripts out of ten from the primary source. A book review and a research report that doesn’t relate to teacher education were excluded. Appendix 1 includes a table showing an overview of the number of articles selected from the primary and secondary bibliographical sources; it also contains the bibliographic details of each of the selected articles. The appendix 1 is available at http://cor.to/pme_OMTE

Analysis of the manuscripts
Once the eighteen manuscripts listed in appendix 1 were selected, we proceeded to analyze them. To carry out the analysis, some guiding questions were defined. These questions were useful to keep the analysis focused on the aspects of the manuscripts that would allow us to answer the research questions.

To try to homogenize the way the guiding questions were interpreted and applied in the analysis of the manuscripts, there was an initial phase in which all the members of the research team independently applied the guiding questions to 3 of the 18 manuscripts contained in appendix 1. After analyzing the articles independently, the members of
the research team met to compare their results. This stage helped to homogenize the interpretation of the guiding questions and the analysis of the manuscripts. After this stage, the researchers continued examining the manuscripts independently, but meeting regularly to share and discuss their results. The guiding questions used to analyze each of the manuscripts were: (1) what is (are) the research question(s) addressed in the study? (2) what technological tools are used to generate empirical data and what kind of data is generated? (3) what methods are used to analyze the empirical data? (4) what theoretical constructs are used in the study? (5) do you notice any transformation in the work of the researcher(s) conducting the study?

The guiding question (1) was designed to obtain information to answer the research question RQ1. The guiding questions (2), (3) and (4) were used to identify information that could allow us to answer the research question RQ2. Particularly, the guiding question (2) was aimed at investigating the kind of empirical data presented in the reviewed studies, and the role of technological resources in the generation of such data; this information allowed us to answer the research question RQ3. The guiding question number (5) was not focused on identifying a particular type of information contained in the manuscripts, it was used as a question that required us to reflect on the possible changes in the work of researchers as addressed in the research question RQ4.

The answers to the guiding questions connected to each of the analyzed papers were written into tables and categorized. In the next section of the manuscript we present the categorizations constructed, which in turn provide answers to our research questions.

RESULTS

The presentation of our results revolves around four aspects: first, we provide an overview of the main topics that have been investigated in this area; second, we mention the main tools used for the collection of empirical information as well as the theoretical constructs that have been used in these investigations; third, we refer to the type of empirical data used in the studies, as well as the techniques employed to analyze them. Finally, we reflect on an issue that relates to the above three aspects: the possible transformations that online environments may produce on the researchers’ work and the type of knowledge they produce.

Main issues investigated in the area of OMTE (answer to RQ1)

The research reports included in this review have a common core feature: all of them focus on aspects of mathematics teachers’ knowledge through the use of online environments. The online environment has played different roles in the research reviewed. In most of the studies it has been used as a means to conduct research, but it is also intended as an element that could be incorporated into mathematics teachers’ work. Also, some research has focused on phenomena that occur as a result of working with mathematics teachers in online environments.

According to the research interests reflected in the reviewed studies, we have constructed a categorization consisting of two groups that aren’t necessarily mutually
exclusive: (a) studies focused on analyzing interactions among teachers in online settings, and (b) studies focused on teachers’ professional development.

(a) Studies focused on analyzing interactions among teachers in online settings

This kind of studies is conducted with groups of pre-service teachers, in-service teachers, and teacher educators. In these studies online-based interactions among teachers are promoted, and then researchers focus on investigating the specific ways in which teachers communicate and interact in such online collaborative environments. For instance, Silverman (2012) explores the relationship between teacher participation in online discussions and the development of their mathematical content knowledge for teaching. For this, social network analysis methods are employed for coding, comparing and categorizing teachers’ participation in online discussions. Schemes such as “Cheerleading/Affirming”, “Doing Mathematics” and “Questioning/Challenging” are used for the coding teachers’ participation. Subsequently, graphical representations of the interactions among teachers are developed with the help of social network analysis software.

(b) Studies focused on teachers’ professional development

Mathematics teachers’ professional development is a recurrent theme in the literature reviewed. Although there are different interpretations of the concept of professional development in the literature, it is generally understood as changes in the teachers that favor improvements in their professional practice. Some research focuses on investigating how the work and involvement of mathematics teachers in online environments promotes their professional development; for instance, Fernández, Llinares & Valls (2012) study how prospective teachers’ participation in online discussions when solving specific tasks, supports the development of their capacity of noticing of students’ mathematical thinking. Clay, Silverman & Fisher (2012) studied how teachers, after participating in online collaborative work, begin to transform their language incorporating elements of a theoretical approach called Learning Algebra with Meaning; the transformation of teachers’ language and its use in analyzing students’ mathematical activity, are considered indicators of professional development.

Main theoretical approaches employed in the area of OMTE (answer to RQ2)

We found that some of the theoretical approaches used are extrapolations into online environments of theoretical tools originally designed for face-to-face settings; examples of this are the concepts of community of practice (used in the study of Kynigos & Kalogeria, 2012) and mathematical knowledge for teaching (Clay et al, 2012). However, theoretical approaches originally designed for online or technologized environments are also employed, for instance the concept of humans-with-media (see Borba, 2012); there are also methodological tools specifically designed for application to online scenarios such as the model of instruction called online asynchronous collaboration and developed by Ellen Clay and Jason Silverman (Clay et al, 2012).
Type of empirical data, how are obtained and analyzed (answer to RQ3)

By empirical data we refer to all kind of data that the researchers have considered as the unit of analysis in their research. Because the research is developed in online settings, the type of data generated is of digital nature; more particularly, the empirical data can be classified as:

Written productions: includes interactions in discussion forums, interviews via e-mail, and discussions in chat rooms. For example, in Fernández, Llinares, & Valls (2012) asynchronous forums are analyzed; in such forums teachers discuss the contents of videos of their students solving problems and also students’ writing assignments.

Teaching materials: in some studies the focus is on the teaching resources designed by mathematics teachers for teaching a particular topic. For example Goos and Geiger (2012) report an study in which prospective teachers are asked to create video presentations along with a set of questions that would engage primary school students in mathematically rich learning. The video material’s potential to encourage a critical perspective on mathematics teaching and learning is studied.

Mathematical productions: in this category we consider studies that focus on studying teacher-mathematical content relationships. For instance, in Borba and Zulato (2010) the geometric constructions performed by teachers when using a geometry software are analyzed.

The ways in which the data are analyzed are diverse; however, in the review we mainly found qualitative studies. Some of these qualitative studies include quantitative analysis, for example Silverman (2012) made a qualitative categorization of online interactions between teachers, but he also uses social network analysis to quantify such interactions. Another example is the study of Meletiou-Mavrotheris (2012) that includes quantitative data such as the number of messages that emits a participating teacher in an online course. The analysis of teachers’ mathematical productions is less common. An example of this is the work of Borba and Zulato (2010) where is analyzed how teachers incorporate a software with graphic capabilities into the process of producing mathematical knowledge.

The main tools used by researchers to generate their empirical data can are graphing software; platform resources such as forums, chat rooms and questionnaires; and digital recording artifacts such as iPods, camcorders, and smartphones.

Is the work of the researcher transformed by the characteristics of the online environments? If yes, how? (answer to RQ4)

The answer to the first question is: yes, through our review we have noticed changes in the work of researchers, which are directly related to the technological tools available in the online environments. To clarify the nature of these changes, we used the concept of humans-with-media as a metaphor that “can lead to insights regarding how the production of knowledge itself takes” (Borba & Villarreal, 2005, p. 23); this is, we focused our attention on the unit researchers-with-online environments to identify
steps in the production of research knowledge which are transformed by the characteristics of the online environments. In particular we have identified three instances of transformation:

*Access to data.* Online environments allow researchers to access remote data and in a less intrusive manner. With access to remote data we refer to overcoming geographical barriers when retrieving data, for instance, there are studies where online interactions of teachers coming from different geographical regions are analyzed, such is the case of the study of Meletiou-Mavrotheris (2012) involving teachers of statistics from three European countries. We speak of a less intrusive access to data because online environments allow researchers to observe interactions, dialogues, and teachers’ mathematical productions without being physically present. This feature provides the researcher with observations that are less intrusive than observations of interactions in a face-to-face setting.

*Data collection and processing.* Online environments may also facilitate and accelerate the collection and processing of data. An example of this is the work of Meletiou-Mavrotheris (2012) where they apply online questionnaires to mathematics teachers and the answers can be quickly captured and processed. In this same study quantitative data on teacher participation in online discussions are used (number of teachers participating in a discussion forum or successfully completing group assignments, number of postings by each participant, etc.), however, these data are automatically generated by the online platform where the discussions take place. This type of data provides researchers with access to features of the interactions and collaboration among teachers that would be difficult to access in face-to-face settings; with these data for instance it is possible to develop detailed studies of interaction patterns within different online discussion groups.

*Adaptation and creation of theoretical tools.* Finally, this review has made us notice that online environments create the need to adapt and create theoretical and methodological constructs adequate to study the didactic phenomena related to OMTE. For instance, Hoyos (2012) refers to the use of the documentational approach to structure teachers’ interactions in online asynchronous forums, in order to promote teachers’ reflection, however this theoretical approach was initially designed to be applied on face-to-face settings.

**References**


